How Many Records Should Be Used in ASCE/SEI-7 Ground Motion Scaling Procedure

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Current building codes in the U.S. refer to ASCE/SEI-7 provisions for selecting and scaling ground motions for use in nonlinear response history analysis of structures. According to the ASCE/SEI-7, if at least seven ground motions are analyzed, the design values of engineering demand parameters (EDPs) are taken as the average of the EDPs determined from the analyses. If fewer than seven ground motions are analyzed, the design values of EDPs are taken as the maximum values of the EDPs. ASCE/SEI-7 requires a minimum of three ground motions. Because these limits in the number of records are based on engineering experience, this study examines the required number of records statistically such that the scaled records provide accurate, efficient and consistent estimates of "true" structural responses. Based on elastic-perfectly plastic and bilinear single-degree-of-freedom systems, the ASCE/SEI-7 scaling procedure is applied to 480 sets of ground motions; the number of records in these sets varies from three to ten. As compared to benchmark responses, it is demonstrated that the ASCE/SEI-7 scaling procedure is conservative if less than seven ground motions are employed. Utilizing seven or more randomly selected records provides more accurate estimate of the EDPs. Selecting records based on their spectral shape and design spectral acceleration increases the accuracy and efficiency of the procedure.

INTRODUCTION

When nonlinear response history analysis (RHA) is required for design verification of building structures, the International Building Code (2006) and California Building Code

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(2007) refer to the ASCE/SEI-7 Section 16-2[*] (ASCE, 2005, 2010). According to these documents, earthquake records should be selected from events of magnitudes, fault distance and source mechanisms that comply with the maximum considered earthquake (MCE).

For two-dimensional analysis of symmetric-plan buildings, ASCE/SEI-7 requires intensity-based scaling of ground motion records using appropriate scale factors so that the mean[†] value of the 5 percent-damped response spectra for the set of scaled records is not less than the design response spectrum over the period range from $0.2T_n$ to $1.5T_n$ (where T_n is the elastic first-"mode" vibration period of the structure). For three-dimensional analyses, ground motions should consist of pairs of appropriate horizontal ground motion acceleration components. For each pair of horizontal components, a square root of the sum of the squares (SRSS) spectrum should be constructed by taking the SRSS of the 5 percent-damped response spectra of the unscaled components. Each pair of motions are then scaled with the same scale factor such that the mean of the SRSS spectra from all horizontal component pairs does not fall below the corresponding ordinate of the target spectrum in the period range from $0.2T_n$ to $1.5T_n$. The design value of an engineering demand parameter (EDP)—member forces, member deformations or story drifts—is taken as the mean value of the EDP over seven (or more) ground motions, or its maximum value over all ground motions, if the system is analyzed for fewer than seven ground motions. This procedure requires a minimum of three records. These limits on the number of ground motions are based on engineering experience rather than a comprehensive evaluation (personal comm. with Charlie Kircher).

This study, for the first time, statistically examines the required number of records for the ASCE/SEI-7 procedure such that the scaled records provide accurate, efficient and consistent estimates of "true" median structural responses. The adjective "accurate" refers to the discrepancy between the "true" responses and those computed from the scaling procedure. The adjective "efficient" refers to the record-to-record (that is, intra-set) variability of responses, and the adjective "consistent" refers to the ground motion set-to-set (that is, inter-set) variability of accuracy and efficiency. Smaller values of inter- and intra-set dispersion of responses indicate that the scaling procedure is more efficient and consistent.

[*] Because the ground motion scaling procedure for two-dimensional analysis of structures is same in ASCE/SEI-7-05 and -7-10 documents; we simply refer to this method as the ASCE/SEI-7 scaling procedure in the remaining of this paper.

[†] In the remaining, "mean" is used in lieu of "arithmetic mean".

Based on elastic-perfectly plastic and bilinear single-degree-of-freedom (SDF) systems, the accuracy, efficiency and consistency of the ASCE/SEI-7 ground motion scaling procedure are evaluated by applying it to 480 sets of ground motions. The number of records in these sets varies from three to ten. The scaled records in each set were selected in three different ways: (i) randomly; (ii) minimizing discrepancy between scaled spectrum of a record and the design spectrum over the period range from $0.2T_n$ to $1.5T_n$ (this approach will be referred to as "Best1"); (iii) minimizing discrepancy between scaled spectrum of a record and the target spectrum over the period range from $0.2T_n$ to $1.5T_n$, and then identifying the final set of records with spectral acceleration values at T_n close to that of the design spectrum (this approach will be referred to as "Best2").

GROUND MOTIONS SELECTED

Thirty records selected for this investigation (listed in Table 1) were recorded from seven shallow crustal earthquakes compatible with the following hazard conditions:

Moment magnitude: M_w=6.7±0.2

Joyner-Boore distance: R_{JB}=25±5 km

NEHRP soil type: C and D

Highest usable period ≥ 4 sec

Shown in Figure 1 are the 5 percent-damped geometric-mean response spectra for the x-component (identified as the maximum horizontal component) of the unscaled ground motions. The geometric-mean spectrum of thirty records is taken as the design spectrum (that is, target spectrum) for purposes of this investigation.

DESCRIPTION OF INELASTIC SDF SYSTEMS

The structures considered are 16 SDF systems with vibration periods equal to 0.2, 0.5, 1 and 2.5 sec, and yield strength reduction factors R equal to 1, 2, 4 and 8. The design base shear is determined as the mass of the system (assumed to be 1 kip-sec^2/in) times the geometric-mean pseudo-acceleration at T_n reduced by R. The damping ratio of the selected SDF systems is 5 percent. The two constitutive models used for the inelastic SDF systems are: (1) an elastic-perfectly-plastic model, and (2) a bilinear model with 10 percent strength hardening ratio.

Table 1. Selected near-fault ground motion records

Record Sequence Number	Earthquake Name	Year	Station Name	Earthquake Magnitude (M_w)	Joyner-Boore Distance (km)	NEHRP Site Class	Highest Usable Period (sec.)
1	San Fernando, Calif.	1971	LA - Hollywood Stor FF	6.6	22.8	D	4
2	San Fernando, Calif.	1971	Santa Felita Dam (Outlet)	6.6	24.7	C	8
3	Imperial Valley (AS), Calif.	1979	Calipatria Fire Station	6.5	23.2	D	8
4	Imperial Valley (AS), Calif.	1979	Delta	6.5	22.0	D	16
5	Imperial Valley (AS), Calif.	1979	El Centro Array #1	6.5	19.8	D	8
6	Imperial Valley (AS), Calif.	1979	El Centro Array #13	6.5	22.0	D	4
7	Imperial Valley (AS), Calif.	1979	Superstition Mtn Camera	6.5	24.6	C	8
8	Irpinia, Italy	1980	Brienza	6.9	22.5	C	4
9	Superstition Hills (AS), Calif.	1987	Wildlife Liquef. Array	6.5	23.9	D	8
10	Loma Prieta, Calif.	1989	Agnews State Hospital	6.9	24.3	D	4
11	Loma Prieta, Calif.	1989	Anderson Dam (Downstream)	6.9	19.9	C	4
12	Loma Prieta, Calif.	1989	Anderson Dam (L Abut)	6.9	19.9	C	8
13	Loma Prieta, Calif.	1989	Coyote Lake Dam (Downst)	6.9	20.4	D	8
14	Loma Prieta, Calif.	1989	Coyote Lake Dam (SW Abut)	6.9	20.0	C	8
15	Loma Prieta, Calif.	1989	Gilroy Array #7	6.9	22.4	D	4
16	Loma Prieta, Calif.	1989	Hollister - SAGO Vault	6.9	29.5	C	8
17	Northridge, Calif.	1994	Castaic - Old Ridge Route	6.7	20.1	C	8
18	Northridge, Calif.	1994	Glendale - Las Palmas	6.7	21.6	C	6
19	Northridge, Calif.	1994	LA - Baldwin Hills	6.7	23.5	D	6
20	Northridge, Calif.	1994	LA - Centinela St	6.7	20.4	D	4
21	Northridge, Calif.	1994	LA - Cypress Ave	6.7	29.0	C	4
22	Northridge, Calif.	1994	LA - Fletcher Dr	6.7	25.7	C	5
23	Northridge, Calif.	1994	LA - N Westmoreland	6.7	23.4	D	4
24	Northridge, Calif.	1994	LA - Pico & Sentous	6.7	27.8	D	5
25	Kobe, Japan	1995	Abeno	6.9	24.9	D	16
26	Kobe, Japan	1995	Kakogawa	6.9	22.5	D	8
27	Kobe, Japan	1995	Morigawachi	6.9	24.8	D	10
28	Kobe, Japan	1995	OSAJ	6.9	21.4	D	16
29	Kobe, Japan	1995	Sakai	6.9	28.1	D	8
30	Kobe, Japan	1995	Yae	6.9	27.8	D	16

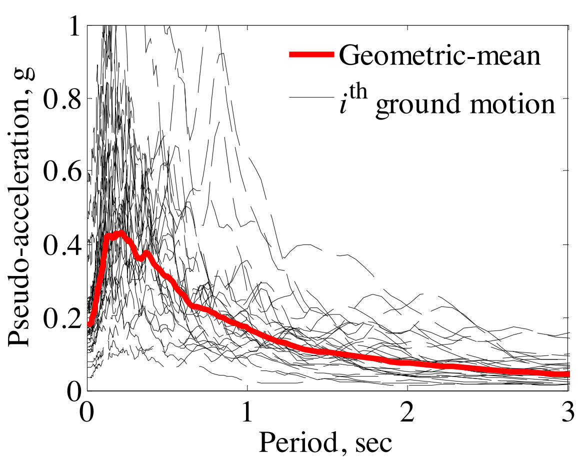

Figure 1. Response spectra of thirty ground motions and their geometric-mean used as the design (that is, "target") spectrum. Damping ratio 5 percent.

METHODOLOGY

According to the ASCE/SEI-7 procedure for two-dimensional (or planar) analyses of "regular" structures, the ground motions should be scaled such that the mean value of the 5 percent-damped response spectra for the set of scaled motions is not less than the design spectrum over the period range from $0.2T_n$ to $1.5T_n$. The ASCE/SEI-7 scaling procedure does not insure a unique scaling factor for each record; obviously, various combinations of scaling factors can be defined to insure that the mean spectrum of scaled records remains above the target spectrum over the specified period range. To achieve the desirable goal of scaling each record with a minimum scale factor closest to unity, we implemented the ASCE/SEI-7 scaling procedure for randomly selected ground motions as in the following:

1. For each of the thirty records listed in Table 1, calculate the 5 percent-damped response spectrum $A(T)$ and the vector $\mathbf{A}$ of spectral values at 300 logarithmically spaced periods T over the period range from $0.2T_n$ to $1.5T_n$.

2. Obtain a target (that is, "design)" pseudo-acceleration spectrum $\hat{A}(T)$ as the geometric-mean spectrum of thirty records. Define $\hat{\mathbf{A}}$ as a vector of target spectral values $\hat{A}_i$ at periods T over the period range from $0.2T_n$ to $1.5T_n$.

3. Compute the scaling factor SF_1 to minimize the difference between the target spectrum $\hat{A}(T)$ (Step 2) and the response spectrum $A(T)$ (Step 1) by solving the following minimization problem for each ground motion $\min_{SF_1}\left\|\hat{\mathbf{A}} - SF_1 \times \mathbf{A}\right\| \Rightarrow SF_1$, where $\|\cdot\|$ is the Euclidean norm. Required for this purpose is a numerical method to minimize scalar functions of one variable; such methods are available in textbooks on numerical optimization (for example, Nocedal and Stephen, 2006). This minimization ensures that each scaled response spectrum is as close as possible to the target spectrum, as shown schematically in Figure 2.

4. Randomly select a set of m ground motions to be used in nonlinear RHA of the systems described previously. No more than two records from the same event should be included in a single set, so that no single event is dominant within a set.

5. Determine the vector $\hat{\mathbf{A}}_{\text{scaled}}$ for the mean scaled spectrum defined as the mean of the scaled spectra ($SF_1 \times \mathbf{A}$) of the set of m records. The ordinates of this mean scaled spectrum could be smaller than the ordinates of the target spectrum at the same periods.

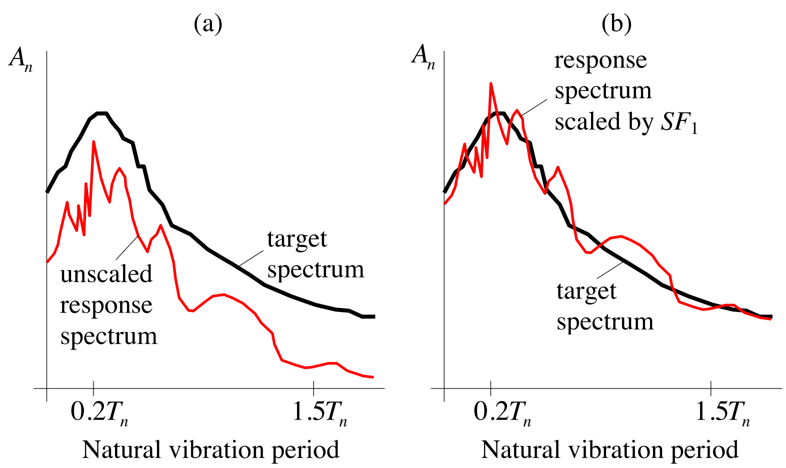

Figure 2. Schematic illustration of Step 3 of the evaluation methodology.

6. Calculate the maximum normalized difference $\varepsilon_{\text{ASCE}}$ (Figure 3a) between the target spectrum $\hat{\mathbf{A}}$ and the mean scaled spectrum $\hat{\mathbf{A}}_{\text{scaled}}$ over the period range from $0.2T_n$ to $1.5T_n$; that is, $\varepsilon_{\text{ASCE}} = \max_{0.2T_1 \le T_i \le 1.5T_1} (\hat{A}_i - \hat{A}_{\text{scaled},i}) \quad \hat{A}_i$, where $\hat{A}_i$ and $\hat{A}_{\text{scaled},i}$ are the ordinates of the target and the mean scaled pseudo-acceleration spectra at vibration period T_i, respectively. Define the scale factor $SF_2 = (1 - \varepsilon_{\text{ASCE}})^{-1}$.

7. Determine the final scale factor $SF = SF_1 \times SF_2$ for each ground motion. Scaling ground motions by the scaling factor SF ensures that the mean value of the response spectra for the set of scaled motions is not less than the target spectrum over the period range from $0.2T_n$ to $1.5T_n$ (Figure 3b).

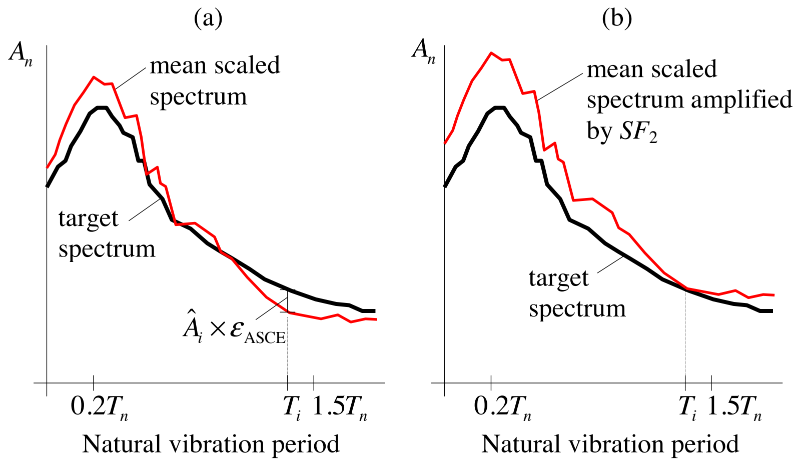

Figure 3. Schematic illustration of Step 6 of the evaluation methodology.

To select ground motions using the approach "Best1", where the discrepancy between the scaled spectrum of a record and the target spectrum over the period range from $0.2T_n$ to $1.5T_n$ is minimized, Step 4 is modified as follows:

4. Rank the scaled records based on their $\left\| \hat{\mathbf{A}} - SF_1 \times \mathbf{A} \right\|$ value; the record with the lowest value is ranked the highest. From the ranked list, select a set of m records to be used in nonlinear RHA of the systems described previously.

Selection of ground motions using the approach "Best2" requires that Steps 4-7 are iteratively implemented until the errors, defined as $\left\| \hat{\mathbf{A}} - SF \times \mathbf{A} \right\|$ and $\left| A(T_n) - SF \times A(T_n) \right|$, are minimized. By executing Steps 1 to 7, the scaling factors for the sets of m ground motions would have been determined. Nonlinear RHA is, then, conducted to obtain final EDP values. If at least seven ground motions are analyzed ($m \geq 7$), the design values of EDPs are taken as the mean or median[‡] of the EDPs over the ground motions used. If fewer than seven ground motions are analyzed, the design values of EDPs are taken as the maximum values of the EDPs.

[‡] Because the geometric mean and median of a lognormal distribution are the same, we decided to employ the term "median" instead of geometric mean, as is commonly done.

BENCHMARK INELASTIC DEFORMATIONS

Benchmark values of inelastic deformations (D_n) were determined by conducting nonlinear RHA of the SDF systems described previously subjected to each of the 30 unscaled hazard compatible ground motions, and computing the median and mean value of the data set.

As mentioned in Hancock et al (2008), the empirical ground-motion models that are used to derive the target spectrum assume the ground motions to be lognormally distributed; therefore, the use of the median response spectrum of the records as a target spectrum is more consistent with the specification of the target spectrum. Similarly, it is commonly assumed that EDPs are lognormally distributed (Cornell et al., 2002); for this reason, it is more appropriate to represent the "mean" structural response by the median; a conclusion that is widely accepted. However, the ASCE/SEI-7 procedure states that the mean values of EDPs are used if at least seven ground motions are considered. Therefore, we decided to use both the median and the mean of the inelastic deformations as the benchmark values. It should be noted that in all cases of benchmark computations, the mean is larger than the median of inelastic deformations, indicating that the distribution of D_n is positively skewed. The percent differences between the two are in the ranges of 15 to 63, 23 to 39, 38 to 48, and 42 to 63 for elastic-perfectly-plastic systems with R equal to 1, 2, 4 and 8, respectively. For bilinear systems the percent differences are 11 to 26, 12 to 32, 20 to 45, and 27 to 56 for the same R values. Note that the difference between mean and median increases with increasing R value.

EVALUATION OF ASCE/SEI-7 SCALING PROCEDURE: LESS THAN SEVEN GROUND MOTIONS

The ASCE/SEI-7 scaling procedure was implemented for the inelastic SDF systems of this investigation subjected to one component of ground motion (Table 1). The accuracy of the ASCE/SEI-7 procedure was evaluated first by comparing the maximum value of the inelastic deformation due to seven sets of 3 to 6 scaled records against the benchmark value, defined as the median (or mean) value of D_n due to the 30 unscaled ground motions. These comparisons are shown in Figures 4 and 5 for bilinear systems with T_n=0.2, 0.5, 1 and 2.5 sec, and R=1, 2, 4 and 8 due to groups of 3, 4, 5 and 6 records called as G3, G4, G5 and G6, respectively. Seven sets of records were considered in each of these groups. Among these seven sets, the first five sets of records were selected randomly out of 30 records, and the two remaining sets of records were selected with the criteria explained previously; these are sets

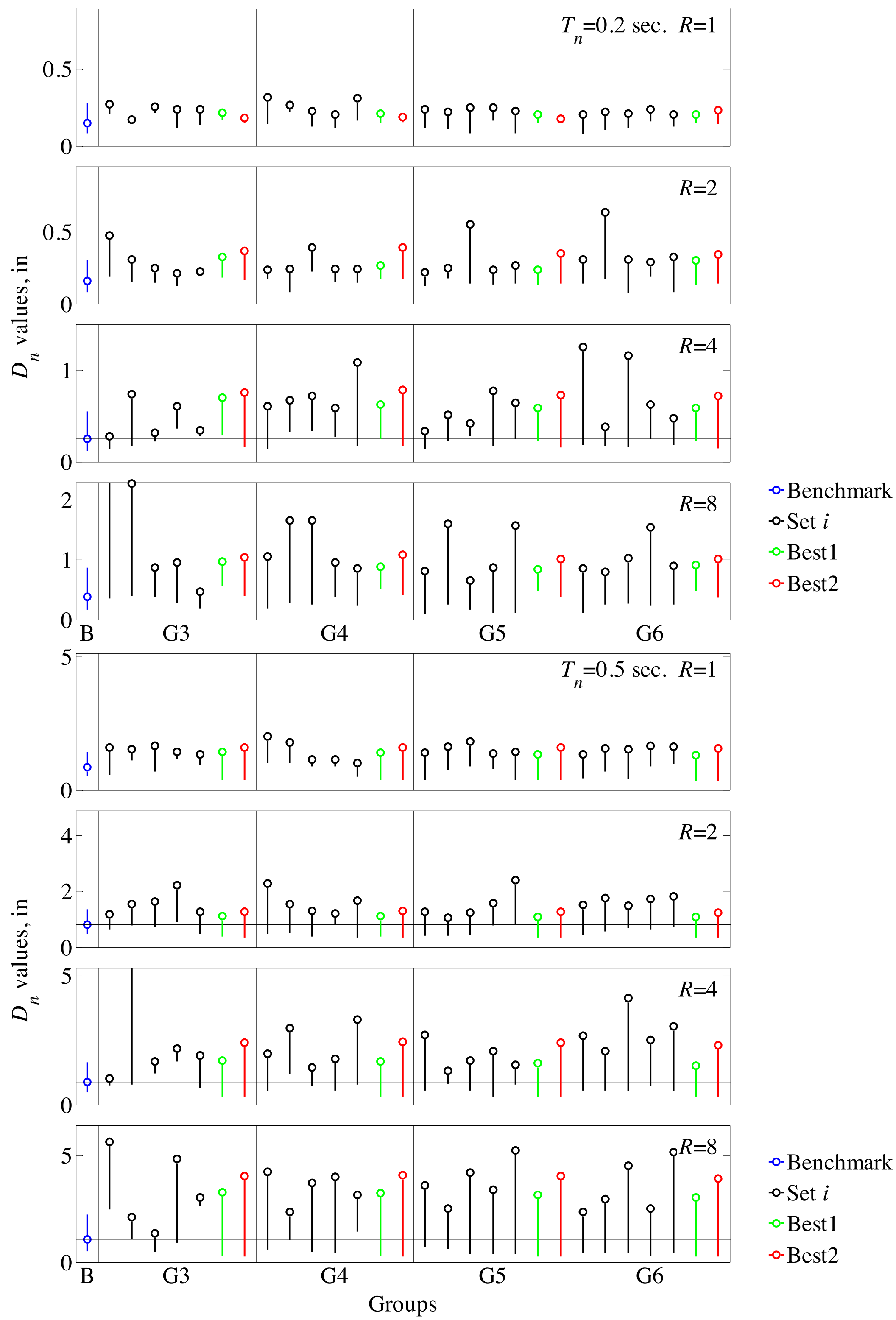

Figure 4. Range of inelastic deformation values for bilinear systems with T_n =0.2 (top panels) and 0.5 sec (bottom panels), and R=1, 2, 4 and 8 for sets of 3, 4, 5 and 6 ground motion records denoted respectively as G3, G4, G5 and G6. The blue dot and the vertical line represent the benchmark (B) median deformation value $\pm\sigma$ assuming a lognormal distribution. For each set, the vertical line and the dot represent the range of the data set and maximum deformation value, respectively.

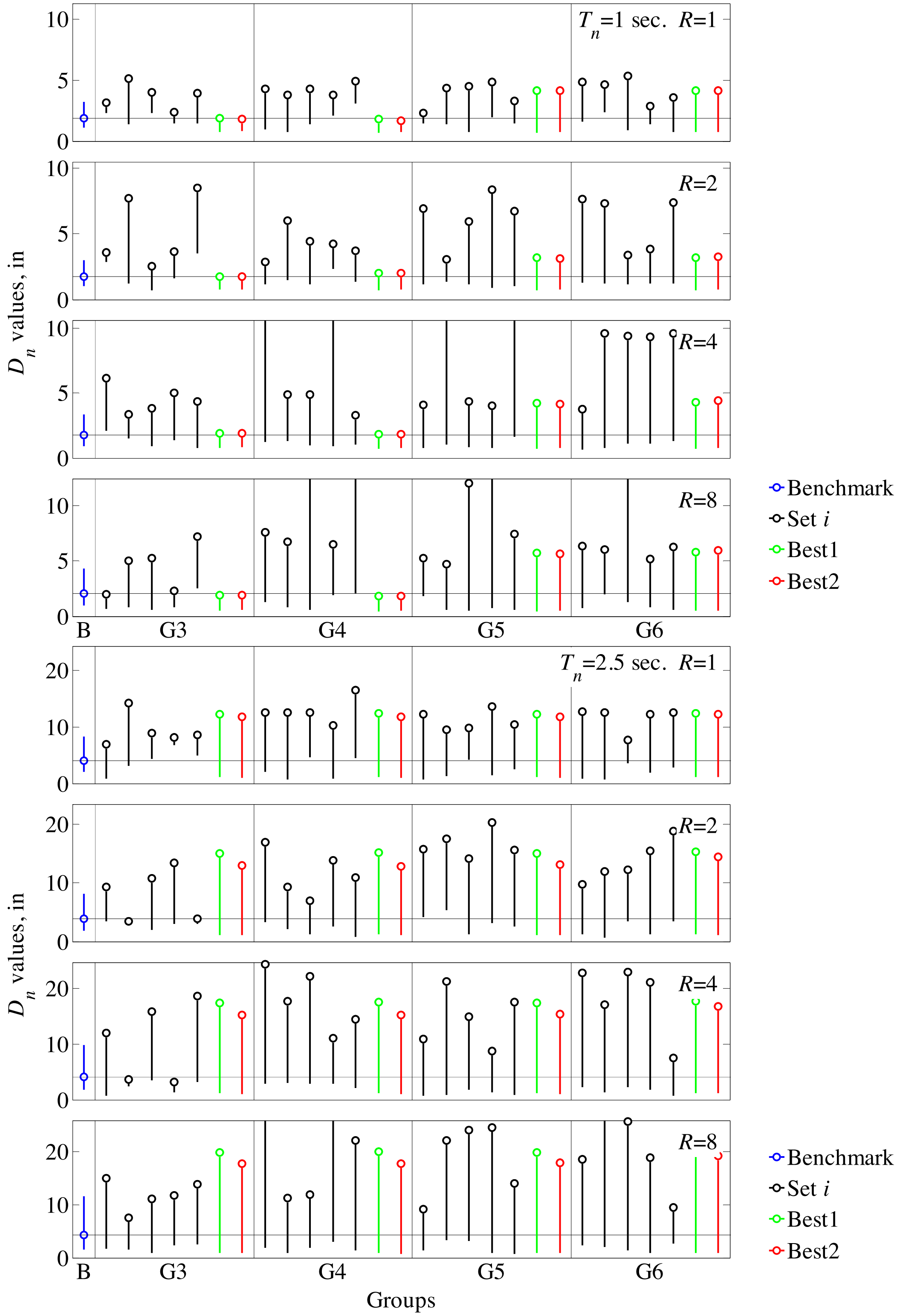

Figure 5. Range of inelastic deformation values for bilinear systems with T_n =1 (top panels) and 2.5 sec (bottom panels), and R=1, 2, 4 and 8 for sets of 3, 4, 5 and 6 ground motion records denoted respectively as G3, G4, G5 and G6. The blue dot and the vertical line represent the benchmark (B) median deformation value $\pm\sigma$ assuming a lognormal distribution. For each set, the vertical line and the dot represent the range of the data set and maximum deformation value, respectively.

"Best1" and "Best2". For each T_n, R, and constitutive model combinations, a total of 30 sets of records are employed. For the benchmark, the blue dot and the vertical line in Figures 4 and 5 represent the median deformation value plus and minus one standard deviation (henceforth denoted as $\pm\sigma$) assuming a lognormal distribution. For each set, the vertical line and the dot represent the range of the data set and maximum deformation value, respectively. Similar plots are presented in Reyes and Kalkan (2011) for elastic-perfectly plastic systems.

Figures 4 and 5 permit the following observations: (1) Increasing the number of records from 3 to 6 has a minor effect in the accuracy of the procedure; overestimations range from 0.4 to 540 percent and from 35 to 680 percent for groups G3 and G6, respectively. (2) The accuracy of the procedure decreases with increasing R value; the maximum error increases from 310 to 750 percent if R changes from 1 to 8. (3) The improvement gained by the use of sets "Best1" and "Best2" is marginal. For R equal 8, the errors range from 10 to 370 percent and from 10 to 350 percent for sets Best1 and Best2, respectively. For elastic-perfectly-plastic systems, the errors are larger than those for bilinear systems (Reyes and Kalkan 2011).

The benchmark results shown in Figures 4 and 5 are based on the median deformation value. Figure 6 compares the benchmark, calculated this time as the mean of 30 D_n values and the ASCE/SEI-7 deformation values for bilinear systems. Included also are the horizontal lines at 0.8 and 1.2 times the benchmark to indicate ±20 percent error around the "true" value. It is apparent that the ASCE/SEI-7 scaling procedure is not accurate and overly conservative as compared to the benchmark results. Insignificant improvement is gained by the use of sets "Best1" and "Best2".

Intra- and inter-set dispersion plots presented in Reyes and Kalkan (2011) show that the intra-set dispersion increases with increasing period, implying that the procedure becomes less efficient. Similarly, inter-set dispersion increases with increasing R, indicating that the procedure becomes less consistent. According to the results presented in Figures 4 through 6, the accuracy, efficiency and consistency in the estimation of inelastic deformations are not achieved in the ASCE/SEI-7 procedure if less than seven records are employed. Therefore the procedure inherently penalizes the analyst for using less than seven records in nonlinear RHAs (personal comm. with Nico Luco).

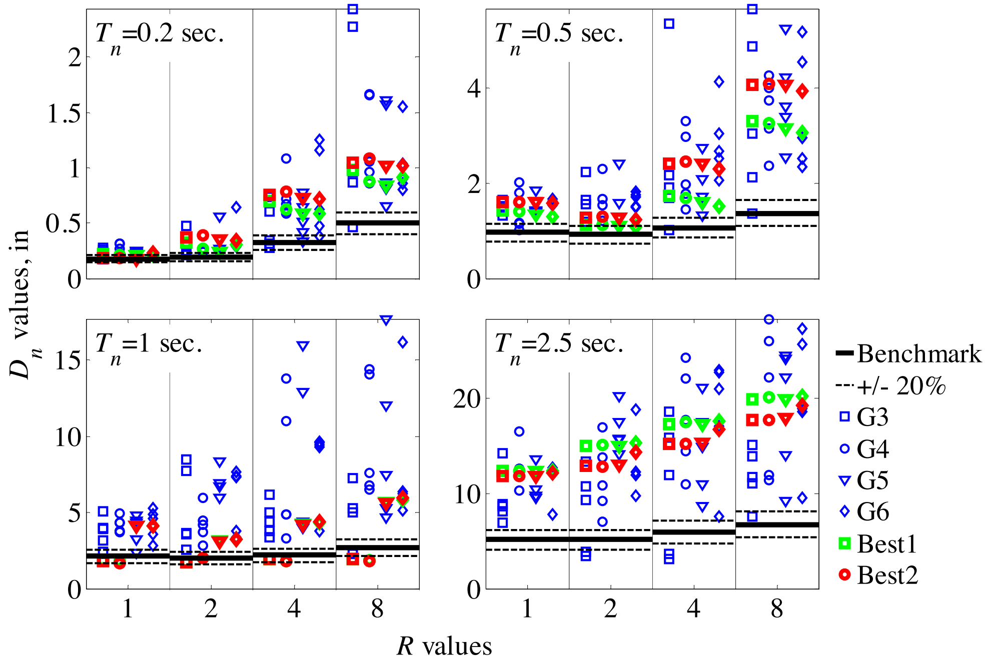

Figure 6. Comparison of benchmark and ASCE/SEI-7 deformation values for bilinear systems. The benchmark EDPs correspond to the mean of 30 deformation values. The deformation values for each set scaled by the ASCE/SEI-7 procedure are obtained as the maximum deformation values of 3, 4, 5 and 6 records in each of seven sets. Included also are sets Best1 and Best2.

EVALUATION OF ASCE/SEI-7 SCALING PROCEDURE: SEVEN OR MORE GROUND MOTIONS

The accuracy of the ASCE/SEI-7 procedure was evaluated next by comparing the median (or mean) value of the inelastic deformation D_n due to seven sets of 7 to 10 scaled records against the benchmark value, defined as the median (or mean) value of D_n due to the 30 unscaled ground motions.

Figures 7 and 8 show the range of inelastic deformation values D_n for bilinear systems due to groups of 7, 8, 9 and 10 records called as G7, G8, G9 and G10, respectively. As explained previously, seven sets were considered in each of these groups (that is, a total of 28 sets plus sets Best1 and Best2 for each T_n, R, and constitutive model combinations). The dot and the vertical line represent the median deformation value $\pm\sigma$ assuming a lognormal distribution. Figures 7 and 8 permit the following observations: (1) Increasing the number of records from 7 to 10 has a minor effect in the accuracy of the procedure; overestimations range from 3 to 155 percent and from 0.6 to 123 percent for groups G7 and G10, respectively. (2) The accuracy of the procedure decreases with increasing R value and

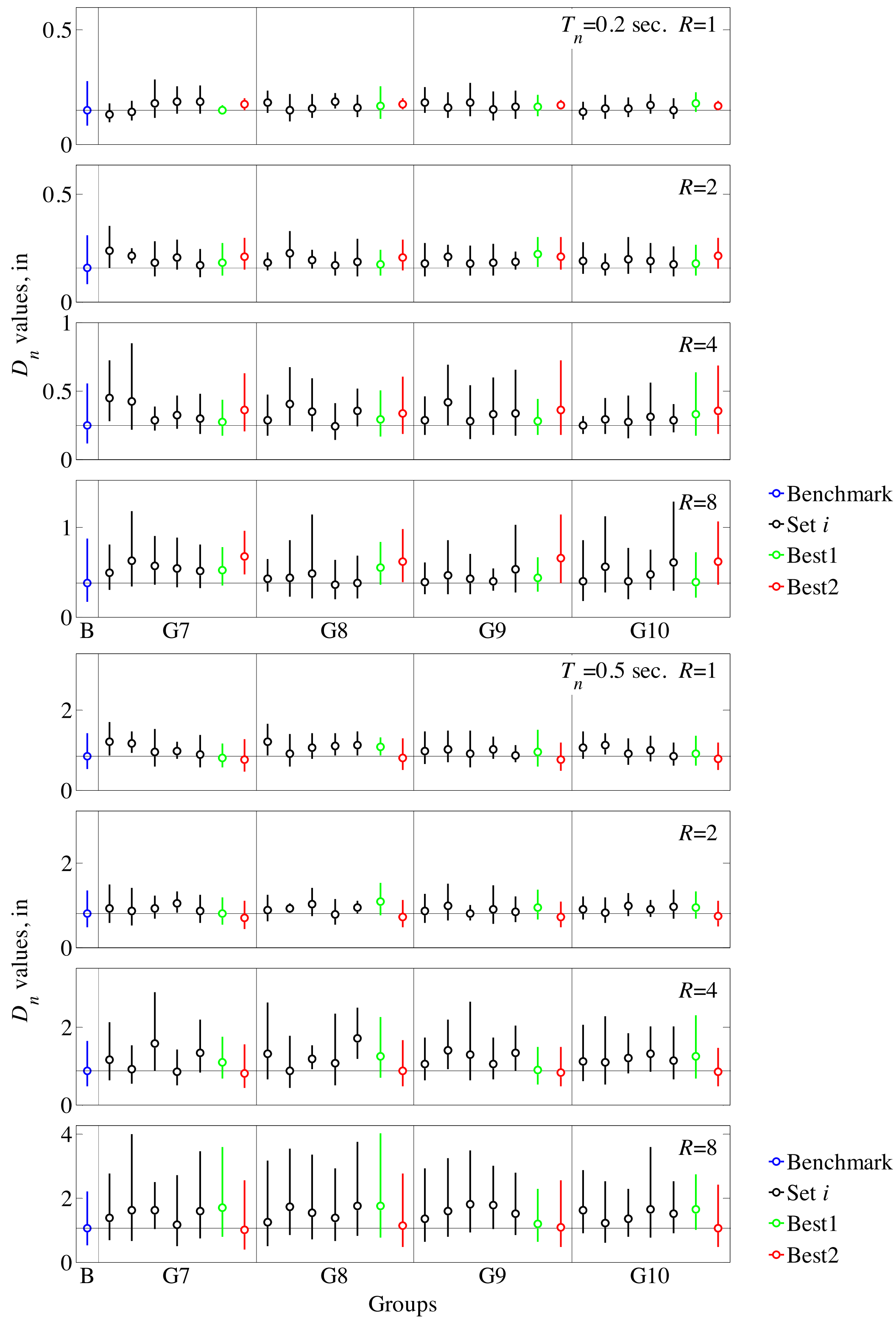

Figure 7. Range of inelastic deformation values for bilinear systems with T_n=0.2 (top panels) and 0.5 sec (bottom panels), and R=1, 2, 4 and 8 for sets of 7, 8, 9 and 10 ground motion records denoted respectively as G7, G8, G9 and G10. The blue dot and the vertical line represent the benchmark (B) median deformation value $\pm\sigma$ assuming a lognormal distribution. For each set, the vertical line and the dot represent the range of the data set and median deformation value, respectively.

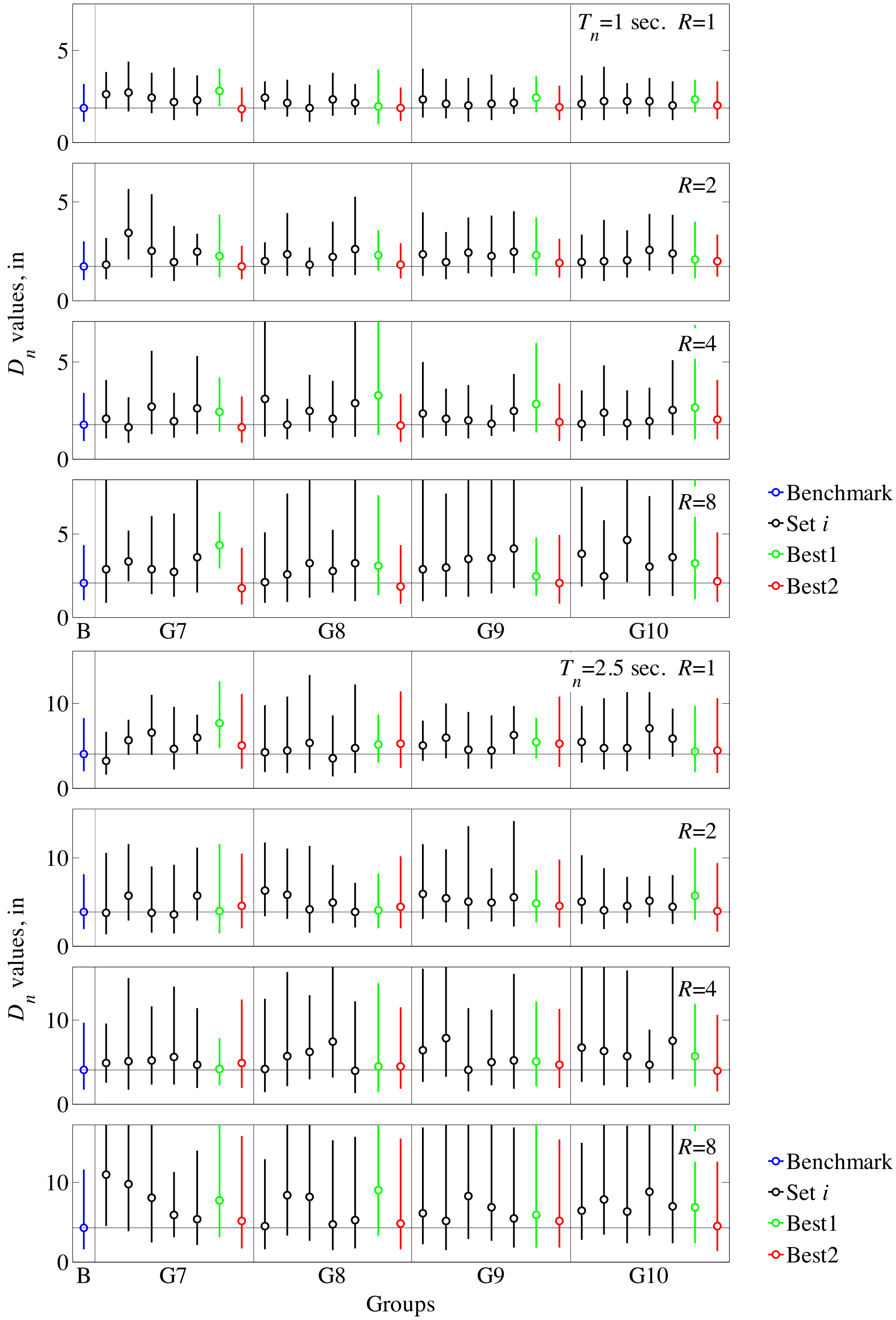

Figure 8. Range of inelastic deformation values for bilinear systems with T_n=1 (top panels) and 2.5 sec (bottom panels), and R=1, 2, 4 and 8 for sets of 7, 8, 9 and 10 ground motion records denoted respectively as G7, G8, G9 and G10. The blue dot and the vertical line represent the benchmark (B) median deformation value $\pm\sigma$ assuming a lognormal distribution. For each set, the vertical line and the dot represent the range of the data set and median deformation value, respectively.

increasing period T_n; the maximum error increases from 74 to 155 percent and from 79 to 155 percent if R changes from 1 to 8, and T_n changes from 0.2 to 2.5 sec, respectively. Figure 9 compares the benchmark results with the ASCE/SEI-7 deformation values for bilinear systems for the range of R values considered; in this case, mean values are used for both the ASCE/SEI-7 and the benchmark results. Included also in this figure are the lines at 0.8 and 1.2 times the benchmark to represent ±20 percent error around the "true" value. By comparing Figure 6 with Figure 9, it is obvious that the ASCE/SEI-7 scaling procedure utilizing seven or more randomly selected records provides more accurate estimate of inelastic deformations. However, the overestimations in median values of inelastic deformation are generally larger than 20 percent, especially for R=4 and 8. Similar conclusions are obtained for elastic-perfectly-plastic systems (Reyes and Kalkan 2011).

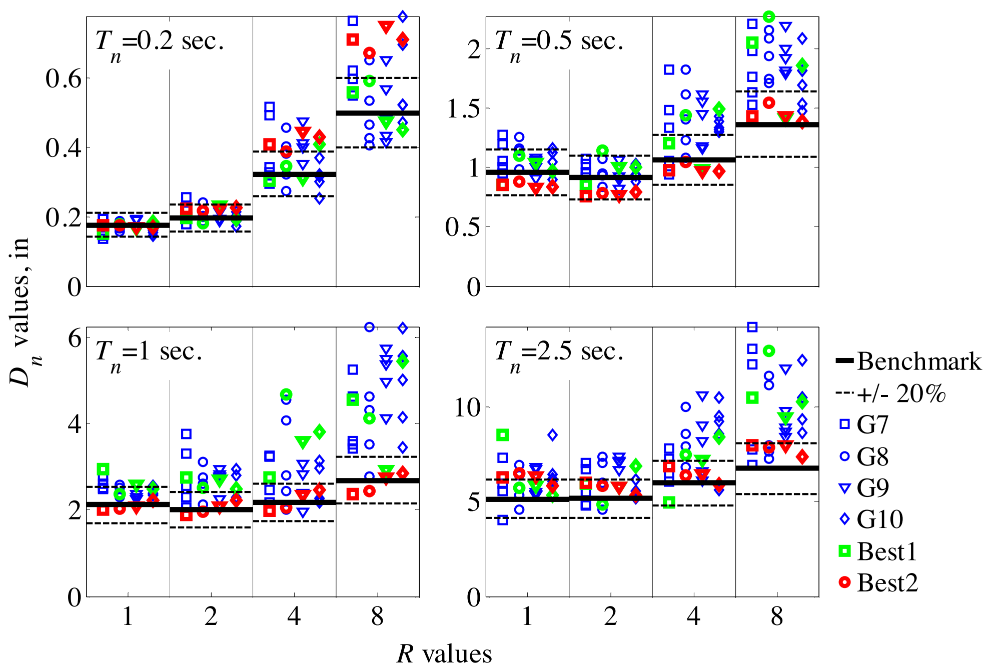

Figure 9. Benchmark and ASCE/SEI-7 deformation values for bilinear systems. The benchmark EDPs correspond to the <u>mean</u> of 30 deformation values. The deformation values for each set scaled by the ASCE/SEI-7 procedure are obtained as the mean of 7, 8, 9 and 10 deformation values. Included also are sets Best1 and Best2.

For systems with short periods and large R values, the mean of randomly selected sets is not similar to the mean of the benchmark data set as demonstrated by the analysis of variance (ANOVA) values in Figure 10. The ANOVA test returns the p value under the null hypothesis that both ASCE/SEI-7 and benchmark results are drawn from populations with the same mean. If p is near zero, it questions the null hypothesis and suggests that the ASCE/SEI-7 mean is significantly different than the benchmark mean. This statistical test indicates that the random selection of records for the ASCE/SEI-7 procedure may lead to inconsistent results.

For systems with $T_n \geq 0.5$ sec or small R values, set "Best2" is much more accurate than set "Best1" demonstrating that consideration of spectral shape and also $A(T_n)$ in selecting and scaling ground motions improves the D_n estimates significantly (Figures 7 through 9). For T_n=2.5 sec, the error of the procedure ranges from 2 to 109 percent and from 1 to 28 percent for sets "Best1" and "Best2", respectively. For systems with very short periods (T_n=0.2 sec) and large R values (4 and 8), both sets "Best 1" and "Best 2" lead to inaccurate estimates of inelastic deformations (Figures 7 through 9); overestimations exceed 40 percent for bilinear systems and 100 percent for elastic-perfectly-plastic systems (Reyes and Kalkan, 2011). This is due to the high variability of spectral pseudo-accelerations and large discrepancies between elastic and inelastic spectra for periods in the acceleration sensitive region and large R values.

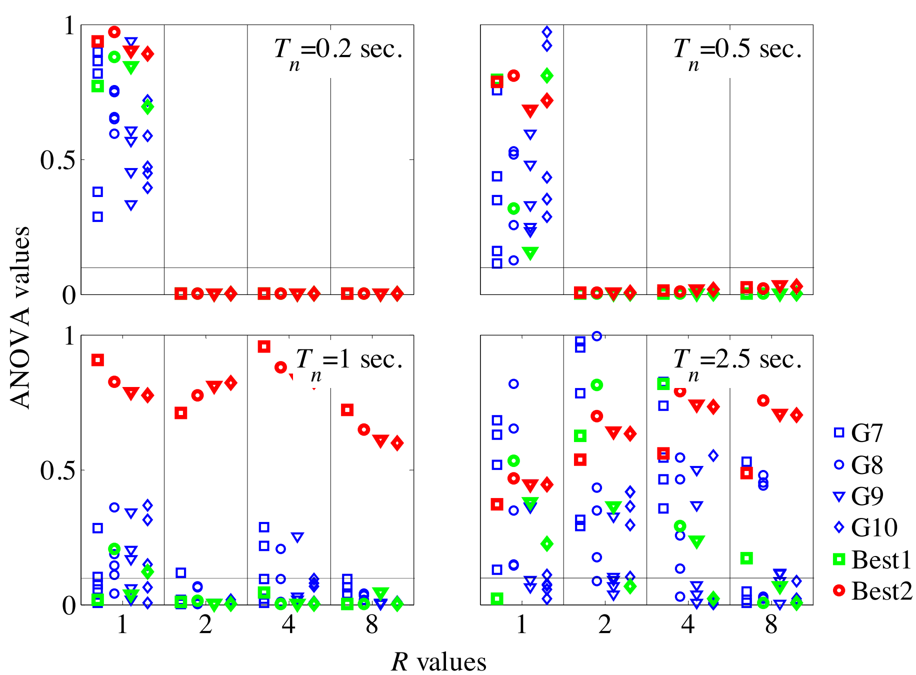

Figure 10. ANOVA values (p) for bilinear systems. The ANOVA test compares deformation values from each set against the benchmark deformation values. Near zero p values suggest that the ASCE/SEI-7 mean is significantly different than the benchmark mean.

The intra- and inter-set dispersion values are shown next in Figures 11 and 12 for bilinear systems, where a lognormal distribution of D_n values was assumed. The intra-set dispersion increases with increasing period, indicating larger variability of response values within a set. Similarly, the inter-set dispersion increases with increasing R values, implying that larger inelastic deformations would lead to increased set-to-set variability. As expected, the inter-set dispersion tends to decrease with increasing number of records per set. Results presented in Reyes and Kalkan 2011 evidence that utilizing seven or more randomly selected records in

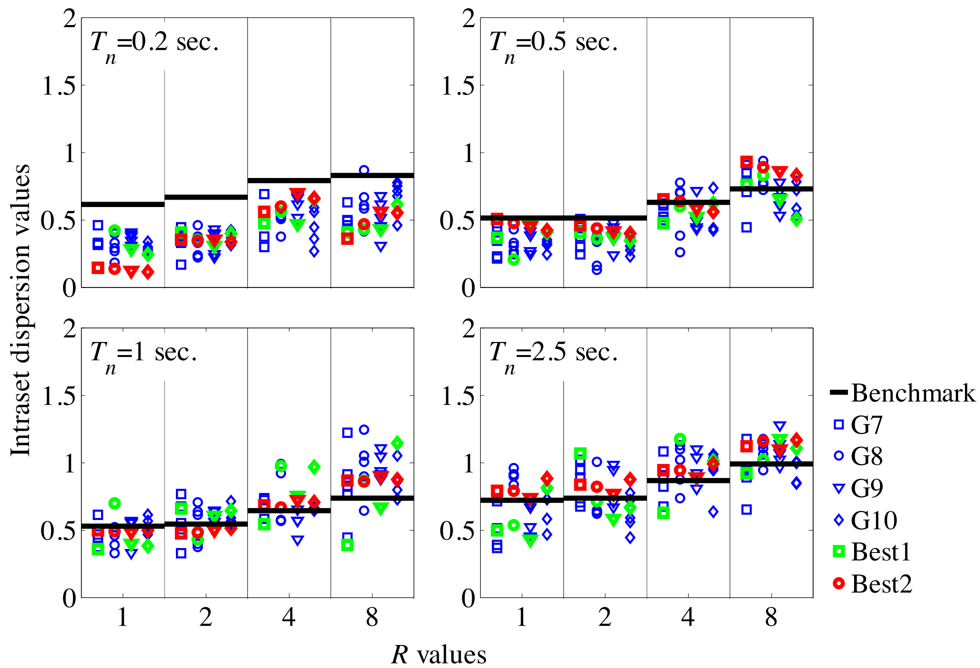

Figure 11. Benchmark and ASCE/SEI-7 intra-set dispersion values for bilinear systems. Lognormal distribution is assumed. Larger intra-set dispersion indicates larger variability of response values within a set; that implies inefficiency.

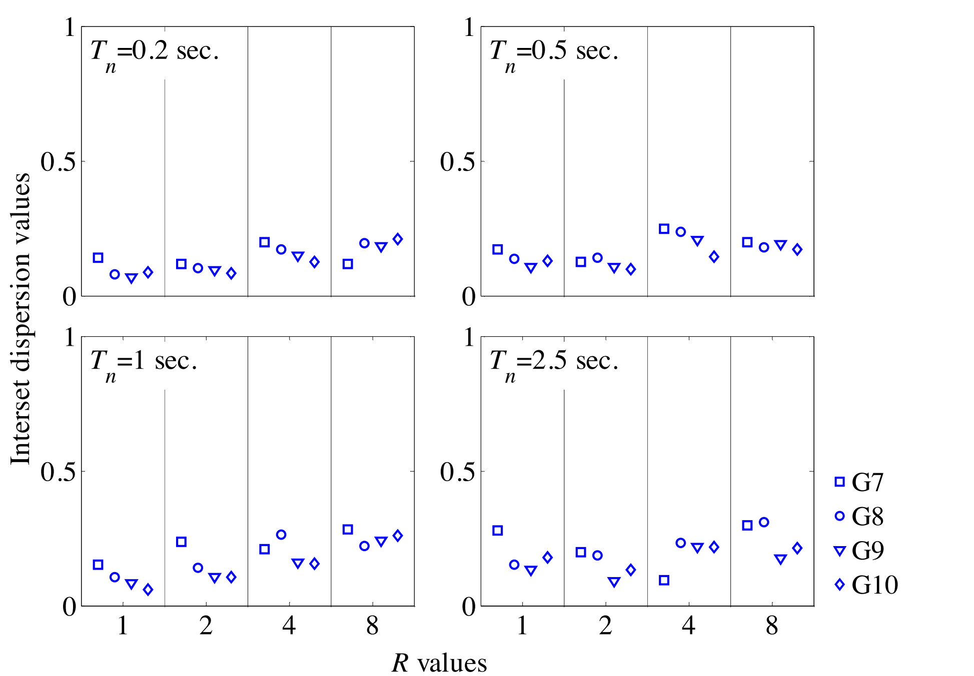

Figure 12. ASCE/SEI-7 inter-set dispersion values for bilinear systems. Lognormal distribution is assumed. Larger inter-set dispersion indicates larger set-to-set variability; that implies inconsistency.

the ASCE/SEI-7 reduces the inter-set dispersion significantly; this reduction is more pronounced for elastic-perfectly-plastic systems. The reduced inter-set variability indicates the consistency in the benchmark estimates of the ASCE/SEI-7 procedure using different sets of records. For systems with a fundamental period in the velocity or displacement sensitive region, accuracy, efficiency and consistency are achieved only if records are selected on the basis of their spectral shape and $A(T_n)$ as opposed to random selection.

MODAL-PUSHOVER-BASED SCALING PROCEDURE: ALTERNATIVE TO THE ASCE/SEI-7 SCALING PROCEDURE

Because the ASCE/SEI-7 ground motion scaling method does not consider explicitly the inelastic behavior of the structure (that is, strength), it may not be appropriate for structures with short periods or for structures located in near-field sites where the inelastic deformation can be significantly larger than the deformation of the corresponding linear system. For such cases, scaling methods that are based on the inelastic deformation spectrum or methods that consider the response of the first-"mode" inelastic SDF system are more appropriate (Luco and Cornell, 2007; Tothong and Cornell, 2008; PEER, 2009). Kalkan and Chopra (2010a-b and 2011a-b) used these concepts to develop a modal pushover-based scaling (MPS) procedure for selecting and scaling earthquake ground motion records in a form convenient for evaluating existing structures and proposed designs of new structures. This procedure explicitly considers structural strength, determined from the first-"mode" pushover curve, and determines a scaling factor for each record to match a target value of the deformation of the first-"mode" inelastic SDF system. If the MPS procedure were applied to the systems of this investigation, it would lead to null error in the estimation of inelastic deformations and null intra- and inter-set dispersions. Therefore, the MPS procedure for SDF systems would be absolutely accurate, efficient and consistent.

CONCLUSIONS

Based on elastic-perfectly-plastic and bilinear inelastic single-degree-of-freedom systems, the accuracy, efficiency and consistency of the ASCE/SEI-7 ground motion scaling procedure are examined by comparing the median and mean values of the inelastic deformation due to 480 sets of scaled records against benchmark results. The number of records in these sets varies from three to ten. The records in each set were selected either (i) randomly, (ii) considering their spectral shapes or (iii) considering the design spectral acceleration value $A(T_n)$ in addition to their spectral shapes. This evaluation of the ASCE/SEI procedure has led to the following conclusions:

1. The ASCE/SEI-7 scaling procedure does not insure a unique scaling factor for each record; obviously, various combinations of scaling factors can be defined to insure that the mean spectrum of scaled records remains above the target spectrum over the specified period range. Utilizing a minimum scale factor closest to unity for each record may overcome this problem.

2. The ASCE/SEI-7 procedure is found to be conservative as compared to the benchmark responses from hazard compatible unscaled records using a larger catalog of ground motions. It is neither efficient nor consistent if less than seven ground motions are utilized, thus penalizing the analyst for employing less than seven ground motions for nonlinear RHAs.

3. The ASCE/SEI-7 scaling procedure utilizing seven or more randomly selected records provides more accurate estimate of inelastic deformations. However, the overestimations in median values of inelastic deformation are generally larger than 20 percent. Increasing the number of records from 7 to 10 has a minor effect in the accuracy of the procedure. Thus, use of 7 records is found to be sufficient.

4. In general, the accuracy of the procedure decreases with increasing R value. The fundamental period T_n (that is, short period or long period systems) does not affect significantly its accuracy if the records are selected randomly.

5. For systems with $T_n \geq 0.5$ sec or small R values (R<4), consideration of spectral shape and also $A(T_n)$ in selecting and scaling ground motions improves the D_n estimates significantly. For bilinear systems with T_n=2.5 sec, the maximum error of the procedure decreases from 109 percent to 28 percent when $A(T_n)$ is considered in addition to spectral shape. For systems with very short periods (0.2 sec) and large R values (4 and 8), however, both sets “Best 1” and “Best 2” lead to inaccurate estimates of inelastic deformations with overestimations exceeding 100 percent for elastic-perfectly-plastic systems and 40 percent for bilinear systems. This is due to the high variability of spectral pseudo-accelerations and the large discrepancies between elastic and inelastic spectra for periods in the acceleration sensitive region and large R

values. For such cases, scaling methods that are based on the inelastic deformation spectrum or that consider the response of the first-“mode” inelastic SDF system are more appropriate.

This study has focused on the statistical examination of the required number of records for the ASCE/SEI ground motion scaling method, which has been limited to elastic-perfectly-plastic and bilinear inelastic single-degree-of-freedom systems.

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NOTATION

The following symbols are used in this paper:

$A(T)$	Pseudo-spectral acceleration at period T
$\mathbf{A}$	Vector of pseudo-spectral acceleration values
$\hat{A}(T)$	Target value of pseudo-spectral acceleration at period T
$\hat{\mathbf{A}}$	Vector of target pseudo-spectral acceleration values
$\hat{\mathbf{A}}_{\text{scaled}}$	Mean scaled spectrum of m records
$\varepsilon_{\text{ASCE}}$	Maximum normalized difference between target and mean scaled spectrum
m	Number of ground motion records
M_w	Moment magnitude
R	Yield-strength reduction factor
R_{JB}	Joyner-Boore distance–perpendicular distance to surface projection of fault plane
SF	Ground motion scaling factor
T_n	Period of single-degree-of-freedom system; or elastic first-"mode" vibration period of structure
V_{S30}	Average shear-wave velocity within 30m depth from surface
σ	Standard deviation